Sl. No. 36

Fuzzy-Stochastic Modelling for Stream Water Quality Management

No. 23/49/2006-R&D

PROJECT COMPLETION REPORT

Submitted to

Indian National Committee on Hydrology (INCOH), Roorkee

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Contents

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3. Title of the Scheme: Fuzzy-Stochastic Modelling for Stream Water Quality Management

4. Financial Details: Pl. see Appendix 1.

5. Original Objectives and Methodology as in the Sanctioned Proposal

(a) Objectives (Reproduced from the sanctioned proposal):

i) To develop a fuzzy-stochastic multi-objective optimization model for the water quality management of a stream. The model would specify, - for known hydraulic characteristics, BOD levels of the industrial and municipal effluents and the environmental constraints on the allowable DO deficits - the optimal treatment levels for the effluents, considering the random variation of the streamflow and conflicting objectives of the industry and the environmental agencies.

The research in the project would aim to enhance the current state of the art in modeling by incorporating the following features:

(a) Use of a sophisticated pollutant transport model (such as the QUAL2E-UNCAS, developed by Environmental Protection Agency, US) in the optimization model, and

(b) Inclusion of randomness of streamflows in the model, thus addressing simultaneously uncertainties due to randomness and fuzziness both in the same modeling framework.

ii) To demonstrate the applicability of the model through a case study in Karnataka state. Interpretation of results, discussion on the implications for industry and the environmental agencies would be provided for the case study.

iii) To document, for the case study application, a step by step procedure for using QUAL2E-UNCAS, including the uncertainty analysis details. This document would act as a user manual for adaptation to any other river system in India. Details of usage of QUAL2E-UNCAS for water quality predictions will also be provided, along with a procedure for estimation of parameters, details of data modules etc., so as to provide guidelines for applications to other river systems.

(b) Methodology Proposed (reproduced from the sanctioned proposal)

A water quality management model generally falls into one of the two broad categories: Models belonging to the first category aim to minimize the waste treatment cost subject to the constraint that water quality standards are not violated at specified water quality check points in the river. Models of the second category maximize the water quality level subject to a constraint on total treatment cost. Since treatment costs are highly nonlinear, uncertain and are difficult to obtain both these approaches pose a difficulty in application. In the proposed study, the concepts of fuzzy set theory will be used to reflect the conflicting goals of the industries and the environmental agencies, avoiding the use of cost functions. Appropriate membership functions will be defined for the purpose. A major advantage of using the fuzzy sets and fuzzy optimization is that the apparently vague and unquantifiable goals may be effectively taken into account in an optimization model. The existing steady state BOD-DO models (e.g., QUAL2E model developed by the EPA) would be used for obtaining the dissolved oxygen concentration at various water quality check points in the river. The BOD-DO model will define a set of constraints in the optimization model. Stochastic nature of the streamflows will be incorporated in the model, with appropriate probability distribution functions. The optimization problem will be formulated as a multi-objective model. A non-linear programming algorithm will be used to solve the model. Uncertainty of streamflows will be incorporated through use of appropriate probability distributions. Fuzzy goals of the industries and the environmental agencies, along with the BOD-DO equations would form the major constraints of the model. The objective function would be formulated to reflect the conflicting nature of the various goals. The model application would be demonstrated through a case study. It is emphasized that only non-reactive pollutants will be considered in the study and no chemistry of the effluent reactions will be involved in the modeling. Only the BOD-DO relationships that are well defined in the literature would be used. The effluent is characterized by its BOD level only. This data is generally available for a given effluent.

The model solution would provide the optimal treatment levels for the effluents, and the associated probability distribution for the DO concentration at various locations along the stream. The solution would be directly useful in determining the amount of the specific industrial effluents - characterized by their BOD levels - that may be discharged into the river at a given location, for a given level of probability of achieving a DO level.

6. Any changes in the objectives during the operation of the scheme:

The objectives were enhanced much beyond those proposed earlier, because a significant progress could be achieved in the project through two master level student theses. Specifically, the following objectives were also achieved, in addition to those proposed originally:

(a) To conduct a First Order Reliability Analysis (FOR A) to identify the key variables and the key locations in the stream that affect significantly the water quality at critical (selected) checkpoints, and

(b) To evaluate the risk of violation of the water quality, with the fractional removal levels prescribed, the FORA and sensitivity analysis.

7. Data Collected and Used in the Analysis, with sources of data:

The project essentially forms a continuing contribution to ongoing research at IISc. A good deal of data has been collected for the case study of Tunga Bhadra river system (pl. see Fig 3, for a schematic of the case study). Specifically, the following data has been collected : Historical streamflow at Tunga-Bhadra junction, historical flows at the four headwaters : Tunga, Bhadra, Kumudavat and Haridra, controlled releases from the Bhadra reservoir into the stream, meteorological data in the catchment, effluent discharge from the eight major dischargers: (a) Municipal discharges from Bhadravati, Shimoga, Davanagere, Harihar and Honhalli, and (b) Industrial discharges from Mysore Paper Mills (MPM), Harihar Polyfibers and Vishvesharaih Iron and Steel Limited (VISL).
The data have been collected mainly from Water Resources Development Organisation (WRDO), Bangalore, Karnataka Pollution Control Board (KPCB) and Bhadravati town municipality. Some data has also been collected from industries.

8. Methodology Actually Followed:

Water quality management problems are characterized by various types of uncertainties at different stages of the decision making process. Uncertainty in water quality management models arises primarily from (i) randomness associated with different input variables of the model, (ii) uncertainty due to the water quality simulation model used, and (iii) imprecision (or fuzziness) associated with the goals of dischargers and the pollution control agency (PCA).
Uncertainty due to randomness of variables and parameters of the river system has received due attention in the development of water quality management models. Major components of the river system that give rise to randomness are the quality and discharge characteristics of both headwater flow as well as effluent flows. These in turn render the water quality indicators (output variables of the water quality simulation model) random in nature. There are three widely adopted approaches for addressing randomness in water quality models (Takyi and Lence 1999). These are (i) chance-constrained optimization (e.g., Ellis 1987); (ii) combined simulation-optimization (e.g., Takyi and Lence 1994);

and (iii) the multiple realization approach (e.g., Burn and Lence 1992; and Takyi and Lence 1999).

The second form of uncertainty, referred to as model uncertainty, arises because of simplifying assumptions used to derive mathematical relations between inputs and outputs in describing a complex process (Tyagi and Haan 2001). Cardwell and Ellis (1993) addressed model uncertainty by simultaneously considering multiple models, such as the Streeter-Phelps (SP) (Streeter and Phelps 1925) equations, QUAL2E (Brown and Barnwell 1987), and WASP4 (Ambrose et al. 1988) for a water quality management problem.

The third form of uncertainty, that due to imprecision, is associated with description of the goals and quantification of *desirable water quality*. In water quality management problems, randomness is not the only relevant uncertainty, imprecision in management goals also has considerable importance. Setting up water quality criteria for any particular use of a water body is an example of uncertainty due to imprecision. A second example is assignment of permissible risk levels for violation of water quality standards. A management model that takes into account uncertainties due to both randomness and fuzziness may be expected to be a more realistic decision making tool for water quality management of river systems.

Efforts have recently been made for simultaneous treatment of randomness and fuzziness in water quality management of river systems. Sasikumar and Mujumdar (2000) have presented a theoretical framework to include both randomness and fuzziness in river water quality management models. The concept of probability of a fuzzy event is used to link probability with fuzzy sets. Methods based on multiple scenarios and optimization (Burn 1989) and the multiple realization method (Takyi and Lence 1999) use Monte-Carlo Simulation (MCS) to generate several possible scenarios of hydrologic, hydraulic, and pollutant-loading conditions. The probability distribution estimated by MCS generally closely approximates the exact one, provided the number of realizations is sufficiently large (Maier at al. 2001). A major disadvantage of MCS, however, is its high computational requirements. To overcome this limitation to a certain extent, First-Order Reliability Analysis (FORA) can be used. FORA, introduced for water quality problems by Burges and Lettenmaier (1975), helps in identifying the combinations of model input parameters and variables that are most likely to result in the failure of the system. It also helps in screening the key checkpoints (i.e., locations with high variability of the water quality indicator) where risk due to uncertainty is likely to be high. In the present study, FORA is used to identify the key variables and key checkpoints in the system, and MCS is applied to obtain the frequency distribution of water quality indicator levels at the key checkpoints with respect to the key input variables. The work demonstrates a procedure for evaluating fuzzy risk using FORA and MCS methods applied to the QUAL2E-UNCAS model (Brown and Barnwell 1987).

The methodology of computing the fuzzy risk is illustrated in Figure 1. The set of optimal fractional removal levels are determined using the fuzzy waste load allocation model (FWLAM). The simulation-optimization approach developed by Mujumdar and Subbarao (2003) is followed for implementing the FWLAM. FORA identifies the key input variables and parameters and also determines the key checkpoints in the river system. The frequency distribution of the water quality indicator levels at the key checkpoints with key variables and parameters treated as random, are obtained from MCS. Appropriate membership functions are assigned to the fuzzy set of *low water quality*. The frequency distribution of the water quality indicator level along with the fuzzy membership functions are then used to evaluate the fuzzy risk of low water quality at the key checkpoints.

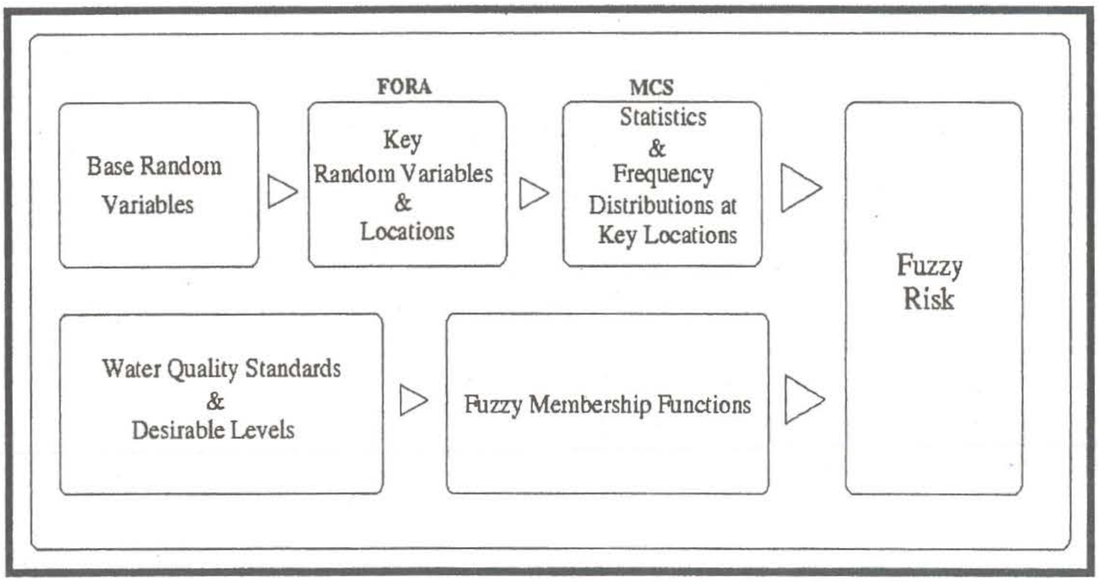

Fig. 1. Evaluation of Fuzzy Risk (Note : FORA : First Order Reliability Analysis; MCS : Monte Carlo Simulation)

8.1 FWLAM

The fuzzy waste load allocation model (FWLAM) developed by Sasikumar and Mujumdar (1998) forms the basis for the optimization model developed in this section. The FWLAM is described using a general river system. The river consists of a set of dischargers who are allowed to release pollutants into the river after removing some fraction of the pollutants. The acceptable water quality condition is ensured by checking the water quality in terms of water quality indicator levels (e.g., DO concentration) at a finite number of locations which are referred to as checkpoints. In a water quality management model, the concentration level of the water quality indicator is expressed as a function of the fractional removal levels for the pollutants released by the dischargers in the river system. An optimization problem is formulated with the set of fractional removal levels and the minimum satisfaction level forming the decision variables. In the FWLAM, the following fuzzy optimization problem is formulated to take into account the fuzzy goals of the PCA and dischargers.

$$Maximize \quad \lambda \tag{1}$$

subject to

$$\mu_{E_{il}}(c_{il}) \geq \lambda \quad \forall\, i, l \tag{2}$$

$$\mu_{F_{imn}}(x_{imn}) \geq \lambda \quad \forall\, i, m, n \tag{3}$$

$$c_{il}^{L} \leq c_{il} \leq c_{il}^{D} \quad \forall\, i, l \tag{4}$$

$$max[x_{imn}^{L}, x_{imn}^{MIN}] \leq x_{imn} \leq min[x_{imn}^{M}, x_{imn}^{MAX}] \quad \forall\, i, m, n \tag{5}$$

$$0 \leq \lambda \leq 1 \tag{6}$$

where c_{il} is the concentration level of water quality indicator i at the checkpoint l of the river system. The PCA sets a desirable level, c_{il}^{D} and a minimum permissible level, c_{il}^{L} for the water quality indicator i at the checkpoint l ($c_{il}^{D} \geq c_{il}^{L}$) which form the bounds on c_{il} as shown in crisp constraint (4). Similarly, x_{imn} is the fractional removal level of the pollutant n from the discharger m to control the water quality indicator i in the river system. The aspiration level and maximum fractional removal level acceptable to the discharger m with respect to x_{imn} are represented as, x_{imn}^{L} and x_{imn}^{M}, respectively. The PCA imposes minimum fractional removal levels that are also expressed as the lower bounds, x_{imn}^{MIN} in constraint (5). The upper bound, x_{imn}^{MAX} of the same constraint represents the technologically feasible maximum fractional removal level. Observing that the maximum acceptable level of pollutant treatment cannot exceed the technologically possible upper limit, x_{imn}^{M} is always considered the upper bound of constraint (5). The fuzzy goal $\mu_{E_{il}}(c_{il})$ in constraint (2) is the goal of the PCA to make the concentration level, c_{il} of the water quality indicator i at the checkpoint l as close as possible to the desirable level, c_{il}^{D}, so that the water quality at the checkpoint l is enhanced with respect to the water quality indicator i, for all i and l. Similarly, $\mu_{F_{imn}}(x_{imn})$ in constraint (3) is the goal of the discharger to make the fractional removal level x_{imn}, as close as possible to the aspiration level, x_{imn}^{L} for all i, m, and n. The membership functions $\mu_{E_{il}}$ and $\mu_{F_{imn}}$ indicate variation of satisfaction levels of the PCA and dischargers with respect to the water quality indicator and fractional removal levels, respectively. The constraints (2) and (3), thus, define the parameter λ as the minimum satisfaction level in the system. The parameter λ also is a decision variable in addition to the set of fractional removal levels, in the optimization problem. Crisp constraints (5) and (6) determine the space of alternatives. It may be noted that if the preferred maximum treatment level of the dischargers x_{imn}^{M} is less than the prescribed minimum x_{imn}^{Min} of the PCA, the solution would be infeasible. This leads to a case of complete conflict between the enforcing agency and dischargers.

Substituting expressions for the membership functions of the fuzzy goals $\mu_{E_{il}}$ and $\mu_{F_{imn}}$ (Sasikumar and Mujumdar 1998) in the constraints (2) and (3), respectively, the mathematical formulation of the fuzzy optimization is written as follows:

$$Maximize \quad \lambda \tag{7}$$

subject to

$$[(c_{il} - c_{il}^{L})/(c_{il}^{D} - c_{il}^{L})]^{\alpha_{il}} \geq \lambda \quad \forall\, i, l \tag{8}$$

$$[(x_{imn}^{M} - x_{imn})/(x_{imn}^{M} - x_{imn}^{L})]^{\beta_{imn}} \geq \lambda \quad \forall\, i, m, n \tag{9}$$

$$c_{il}^{L} \leq c_{il} \leq c_{il}^{D} \quad \forall\, i, l \tag{10}$$

$$\max[x_{imn}^{L}, x_{imn}^{MIN}] \leq x_{imn} \leq x_{imn}^{M} \quad \forall\, i, m, n \qquad (11)$$

$$0 \leq \lambda \leq 1 \qquad (12)$$

The exponents, α_{il} and β_{imn}, appearing in constraints (8) and (9), respectively, are non-zero positive real numbers. Assignment of numerical values to these exponents is subject to the desired shape of the membership functions and may be chosen appropriately by the decision maker. The concentration of water quality indicator c_{il} in constraints (8) and (10) is determined using a water quality simulation model. In this study, the water quality simulation model QUAL2E, developed by the US Environmental Protection Agency (EPA) (Brown and Barnwell 1987) is used to estimate c_{il}. Inclusion of QUAL2E as the simulation model and the presence of the exponents, α_{il} and β_{imn}, appearing in the constraints (8) and (9) render the optimization problem non-linear. The Genetic Algorithm (GA), which is known for efficiently achieving global or near global solutions, is chosen as the optimization tool to solve for the decision variables x_{imn} and λ. The optimization problem is solved with a simulation-optimization (S-O) approach. Since the GA is an unconstrained optimization technique, it is complemented with the *Homomorphous Mapping* (HM) (Koziel and Michalewicz 1999) method to handle the

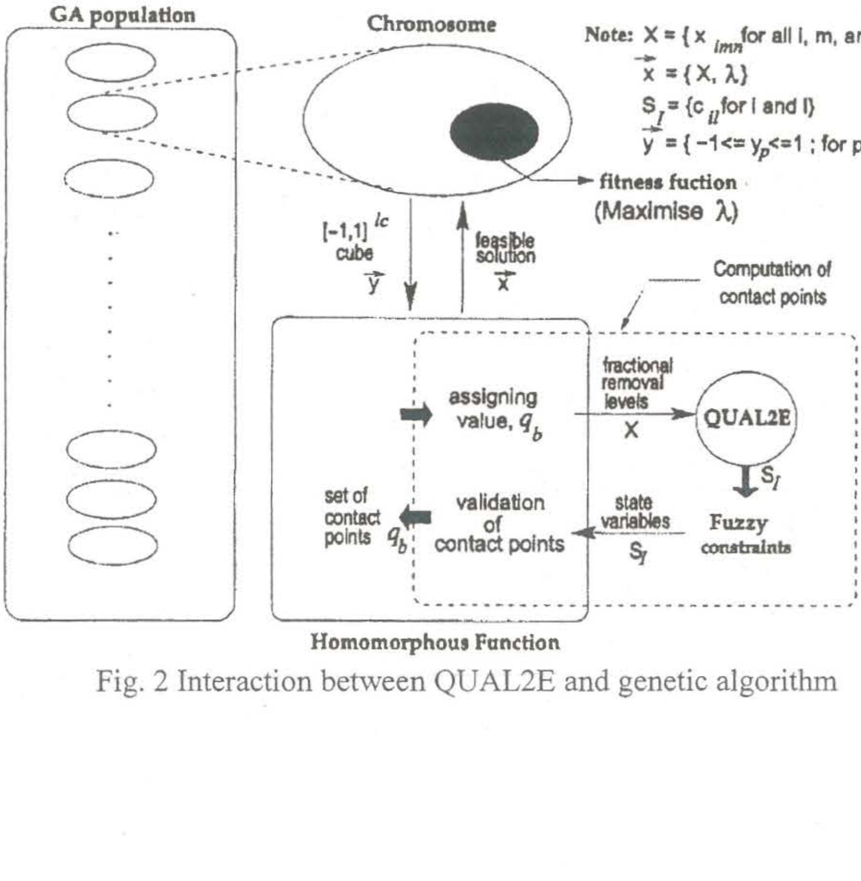

Fig. 2 Interaction between QUAL2E and genetic algorithm

constraints. Interaction among GA, HM, and QUAL2E is shown in Figure 2. Each chromosome of the GA is designed to represent the set of fractional removal levels, x_{imn}, and the satisfaction level, λ, which are all decision variables of the optimization problem. The chromosome is coded in $[-1,1]^{lc}$ (where lc is the length of the chromosome) cube. It means that each element of a chromosome (which is called a *gene*) represents a real number between -1 and 1. HM, after having multiple interactions with the simulation model QUAL2E, maps the $[-1,1]^{lc}$ cube to a feasible solution whose fitness function then is found. In the present case the decision variable, λ, acts as the fitness function for the chromosome. A similar procedure is followed for evaluating the fitness functions of all the chromosomes of the population. After evaluating the fitness functions, the GA applies the operators - reproduction, crossover, and mutation - to generate a new population with improved solutions. The procedure of fitness function evaluation of all the solutions in the new generation is repeated using QUAL2E. In a similar way, GA, HM, and QUAL2E are conjunctively used over the generations until the global solution criteria are met. The solution corresponding to the highest fitness function in the last generation is taken as the optimal solution with the objective function value equal to the fitness function. Again a fixed number of iterations are performed with a different set of GA parameters. In a similar way, different runs are made with different parameter sets to validate the optimal solution. The maximum of all such optimal solutions obtained after performing various runs is taken as the final solution for the purpose of risk evaluation.

A simulation run of QUAL2E with respect to the optimal solution yields the spatial distribution of the water quality indicator level, c_{il}. The checkpoints having the critial water quality indicator levels are chosen for evaluating the fuzzy risk of low water quality in the river system.

8.2 Fuzzy Risk of Low Water Quality

The conventional water quality criteria at checkpoint l is such that any concentration of the water quality indicator less than a specified value, say, c_{il}^{L}, corresponds to a low water quality. This leads to a very stringent definition of low water quality. To overcome this limitation and to account for imprecision in the description of low water quality, Sasikumar and Mujumdar (2000) and Mujumdar and Sasikumar (2002) have introduced a fuzzy set based definition in place of the crisp set based definition of low water quality. The set of concentration levels corresponding to the low water quality is defined as a fuzzy set, W_{il}. Each concentration level in the fuzzy set, W_{il}, is assigned a membership value that lies in the closed interval $[0,1]$. Mathematically, the fuzzy set, W_{il}, is expressed as follows:

$$W_{il} = c_{il} : 0 \leq \mu_{W_{il}}(c_{il}) \leq 1 \tag{13}$$

The membership value, $\mu_{W_{il}}(c_{il})$, of the fuzzy set, W_{il}, indicates the degree of compatibility of the concentration level with the notion of low water quality. The fuzzy risk of low water quality is defined as the probability of occurance of the fuzzy event of low water quality. Mathematically, this can be stated as follows:

$$\textit{fuzzy risk} = P\,(\textit{fuzzy event of low water quality}) \tag{14}$$

$$= \widetilde{P}\ (\textit{low water quality}) \tag{15}$$

where $\widetilde{P}$ denotes the probability of a fuzzy event. The fuzzy risk is computed as,

$$r_{il} = \int_0^{c_{max_{il}}} \mu_{W_{il}}(c_{il})\, f(c_{il})\, dc_{il} \tag{16}$$

where $\mu_{W_{il}}(c_{il})$ is the membership function of the fuzzy set W_{il} of low water quality, $c_{max_{il}}$ is the maximum concentration level, and $f(c_{il})$ is the probability density function (PDF) of the concentration of water quality indicator i at the checkpoint l in the river system. The fuzzy risk of low water quality at a checkpoint indicates the expected degree of low water quality and is a more general form of the crisp risk that indicates the probability of occurance of a low water quality event.

The fuzzy risk defined in Equation (16) is determined at key locations of the river. Sensitivity Analysis and FORA assist in identifying the key locations and key random variables influencing the uncertainty of the model.

8.3 First-Order Reliability Analysis

Reliability-analysis methods like First-Order Reliability Analysis (FORA) and Monte-Carlo Simulation (MCS) are based on multiple simulations and account for the combined effects of parameter sensitivity and parameter uncertainty in the identification of key input variables affecting the uncertainty of the model (Melching and Yoon 1996). An advantage of FORA over MCS is that, for suitable problems, it demands much less computational effort than MCS. However, generally, the probability estimated by MCS approximates the exact value more closely as compared to other methods (e.g., Maier et al. 2001). Considering these aspects, FORA is used for identifying the key variables, whereas MCS is used to obtain the frequency distribution of c_{il}. However, MCS could be done without FORA for prediction of the uncertainty. FORA is used to provide quick evaluation of key parameters and locations.

FORA uses a first-order approximation of the relation between input and output variables for computing variances in multivariate situations. In FORA, a Taylor series expansion of the simulation model output is truncated after the first-order term (Melching and Yoon 1996).

$$y_v = G(X_{ue}) + \sum_{u=1}^{N_b} (x_u - x_{ue})\,(\partial G / \partial x_u)_{X_{ue}} \qquad (17)$$

where y_v is the concentration of the constituent simulated in the selected water quality model; $G()$ is the functional representation of the procedures simulating the constituent. $G()$ may be mass balance equation which forms the basis for the QUAL2E water quality model; X_{ue} is the vector of uncertain basic variables (e.g., model input variables, model parameters, etc.) representing the expansion point; X_u is the vector of uncertain basic variables; and N_b is the number of basic variables.

In FORA applications to water quality management problems, the expansion point is commonly assumed to be the mean value or some other convenient central value of the basic variables. For non-linear systems, this assumption may lead to inaccurate estimation of mean and variance of the model output variable. Also, FORA is normally limited to problems where the random variables have relatively low variance (e.g., CV of less than about 25%). As FORA is applied in the present work only for identifying the key basic variables of the system, but not for quantifying the uncertainty of model output variables, this limitation of FORA may not be very serious in this case. Thus the expected value and variance of the model output, when the expansion point is considered at the mean value of the variables, are:

$$E[y_v] \approx G(X_{uM}) \tag{18}$$

$$var(y_v) = \sigma_{y_v}^2 \approx \sum_{u_a=1}^{N_b} \sum_{u_b=1}^{N_b} (\partial G/\partial x_{u_a})_{X_{uM}} (\partial G/\partial x_{u_b})_{X_{uM}} E[(x_{u_a} - x_{u_aM})(x_{u_b} - x_{u_bM})]$$

where σ_{y_v} is the standard deviation of y_v; and X_{uM} is the vector of mean values of the basic variables. In the above equation, subscripts u_a *and* $u_b \in u$. If the basic variables are statistically independent and derivatives are computed numerically, then

$$var(y_v) = \sigma_{y_v}^2 \approx \sum_{u=1}^{nb} [(\Delta G/\Delta x_u)_{X_{uM}}^2 \quad var(x_u)] \tag{19}$$

The normalized sensitivity coefficient (NSC) which represents the percentage change in the output variable resulting from a unit percentage change in each input variable is computed as follows (Brown and Barnwell 1987; Melching and Yoon 1996),

$$S_{uv} = (\Delta y_v / y_v) \,/\, (\Delta x_u / x_u) \tag{20}$$

where S_{uv}=normalized sensitivity coefficient for output y_v to input x_u ; x_u=base value of input variable [e.g., flow, DO, biological oxygen demand (BOD) values of headwater]; Δx_u=magnitude of input perturbation; y_v=base value of output variables (e.g., DO of the river); and Δy_v =sensitivity of the output variable. The ranking of NSC helps in identifying the key variables affecting the output variable of the river system.

FORA has been successfully applied to water quality models (Burges and Lettenmaier 1975; Chadderton et al. 1982; and Melching and Anmangandla 1992), despite conceptual problems like the assumption of linearity in the functional approximation. FORA is performed initially at a few locations (or checkpoints), where the model output variables are likely to have a significant variability in magnitude. The key locations are identified based on the variance of the output variables at all the locations. Monte Carlo Simulation (MCS) is then applied at the key locations with key variables as the input random variables to obtain the frequency distribution and statistical parameters of the estimated output variable. The source of water in the river system includes distributed flow (or incremental flow) in addition to the headwater and point load flows. The distributed flow addition to the river may be due to runoff from predominantly either agricultural or forest areas and will accordingly affect the water quality in the river. In this work, the fuzzy risk is computed for the case study with and without nonpoint source pollution through incremental flow and the contribution of incremental flow to the fuzzy risk is discussed.

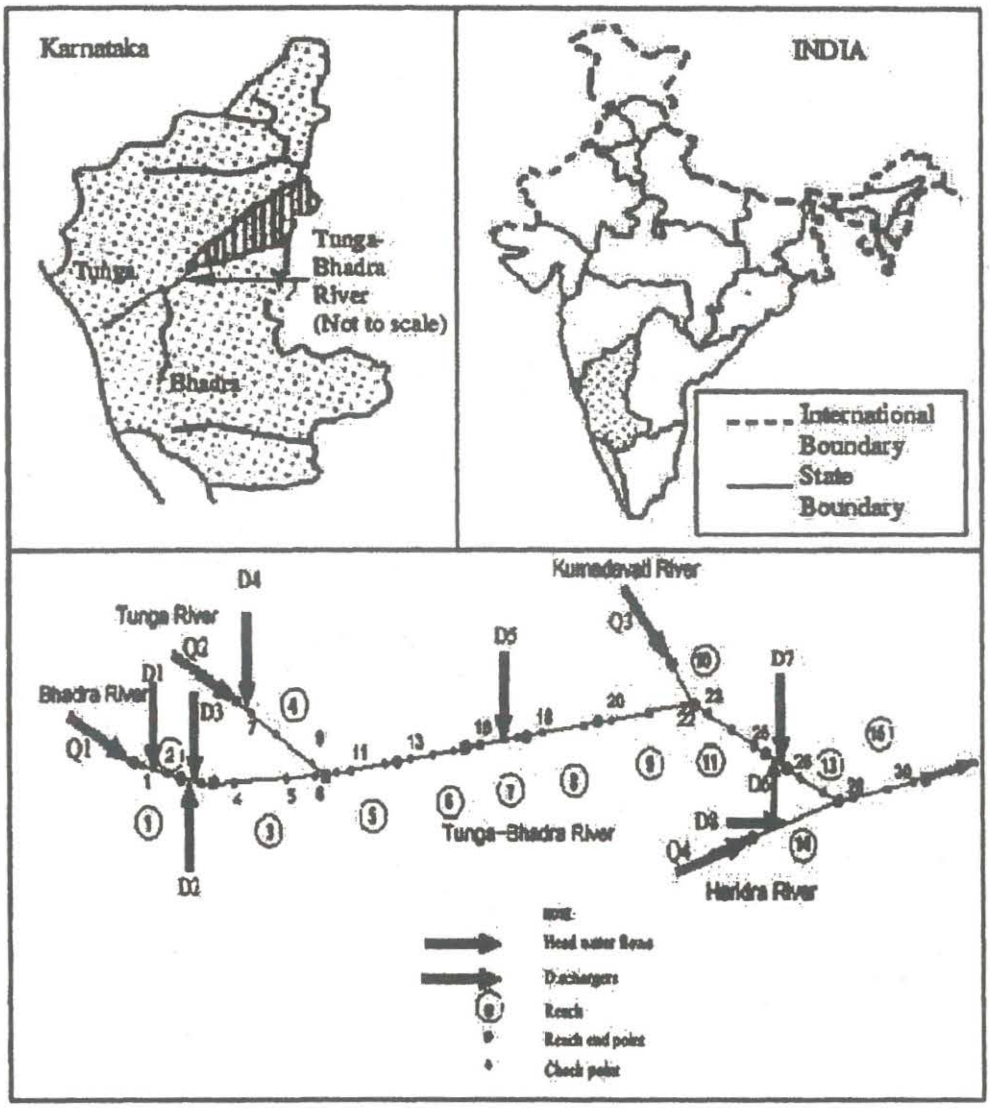

Fig. 3 Location map and schematic diagram of Tunga–Bhadra river system

8.4 Model Application

Application of FORA to the fuzzy waste load allocation model is illustrated through a case-study of Tunga-Bhadra River system shown schematically in Figure 3. The Tunga-Bhadra River is a perennial river formed by the confluence of Tunga and Bhadra Rivers, both tributaries of the Krishna River, in southern India. The river has two other tributaries, the Kumadavati and Haridra Rivers. The river receives the waste loads from eight major effluent points which include five industrial effluents and three municipal effluents. The model is applied to a river stretch of 180 *km* that comprises the four headwaters (Tunga, Bhadra, Kumadavati, and Haridra) and eight point loads (five industrial and three municipal effluents). To keep emphasis on simultaneous treatment of randomness and imprecision, the example is kept simple by considering only one water quality indicator, the DO.

FORA and MCS are implemented using the uncertainty version of QUAL2E, viz., QUAL2E-UNCAS (Brown and Barnwell 1987). Some recent work that applied uncertainty analysis to QUAL2E includes that of Melching and Yoon (1996) in which QUAL2E was used to determine the data required for reducing the model-prediction uncertainty in a water quality model, and of Han et al. (2001) where a reliability model was developed modifying QUAL2E-UNCAS for stochastic water quality analysis of downstream reaches of a river in Korea. Some limitations of the QUAL2E-UNCAS model are that it does not incorporate the effect of periphyton production on DO, and it does not consider non-point or diffuse sources of nutrients or oxygen demanding sources (Maier et al. 2001).

Table 1 Uncertainty information of basic variables

Generic group	Basic variable	Coefficient of variation (%)	Source of data
Temperature coefficients	Biological oxygen demand decay	3.0	a
	Dissolved oxygen aeration	3.0	a
Hydraulic variables	Dispersion correction constant	50.0	a
	Manning's roughness	5.0	a
	Side slopes	5.0	a
	Bottom width	5.0	a
	Slope of channel	5.0	a
Reaction coefficients	Chemical biological oxygen demand oxidation rate	30.0	b
	Reaeration rate	50.0	b
Initial temperature	Initial temperature	10.0	c
Incremental flow	Discharge	27.0	c
	Temperature	8.0	c
	Dissolved oxygen	5.0	a
	Biological oxygen demand	10.0	a
Headwater flow	Discharge	27.0	c
	Temperature	8.0	c
	Dissolved oxygen	5.0	c
	Biological oxygen demand	10.0	c
Point load flow	Discharge	22.4	c
	Temperature	8.0	c
	Dissolved oxygen	5.0	a
	Biological oxygen demand	10.0	a

[a]QUAL2E-UNCAS manual.

[b]Melching and Yoon (1996).

[c]Historical data.

8.4.1 Data Selection

The most important aspect of applying reliability-analysis methods, viz., FORA and MCS, for assessing the statistical parameters of DO concentration is to characterize the uncertainty in the individual input variables required for the QUAL2E-UNCAS model. In

QUAL2E-UNCAS, the uncertainty information is provided in two forms: (i) the coefficient of variation or relative standard deviation (CV), and (ii) the specification of the PDF for each input variable. Table 1 presents the uncertainty information that is used in the present uncertainty analysis (UA). The data of the Table 1 includes the list of the basic variables (both natural variables and model parameters) considered in the analysis, with their CV values, and the source of the CV data. The historical data of the Tunga-Bhadra River system provides information on discharge and temperature characteristics of river and point load flows. 22 years of mean annual flow discharge data obtained from daily flow records of a government agency (Water Resources Development Organization, Bangalore) are used in arriving at the CV of the headwater flow. The value of incremental flow is calculated based on the guage stations located in Bhadra (Reach 1), Tunga (Reach 4) and Tunga-Bhadra (Reach 7) Rivers. Difference between flow at Tunga-Bhadra guage station and sum of the flows at the Bhadra and Tunga guage stations is the flow incremented distributively. The ratio of this difference to the length between the guage stations gives the distributed flow per unit distance, which is 0.34 $m^3/s/m$ in the present case. This value is used as incremental flow throughout the river stretch, to account for the nonpoint source pollution due to runoff. In Indian situations, agricultural sources form only a part of, but do not dominate, the contribution from diffuse pollution in case of BOD (Agrawal, 1999). The nonpoint BOD pollutant is primarily contributed by rural communities, animal husbandry and on-stream activities in India. To conservatively account for uncertainty arising out of lack of adequate data on nonpoint source pollution in the present case, a high value of 30 mg/L for BOD and a low value of 4 mg/L for DO are used for the incremental flow in the analysis. The CV of the incremental flow, BOD and DO are assumed same as those of headwater flow. The CV values of the temperature coefficient, DO, BOD, and all hydraulic variable data except reaeration and deoxygenation rate coefficients are selected from the *typical* range (the other two ranges being *low* and *high*) for QUAL2E-UNCAS applications in Brown and Barnwell (1987, p. 86). The CV values of CBOD (carbonaceous BOD)and deoxygenation rate are obtained from Melching and Yoon (1996).The CVs of point, headwater and incremental inflow discharge and temperature were determined from historic data. Based on the literature (Melching and Yoon 1996), all the input variables except headwater flow are assumed to follow a normal distribution for the purpose of analysis. A log-normal distribution is used for the headwater flow.

In the analysis, input variables are assumed to be uncorrelated, because of lack of adequate data to estimate the correlation structure. If the variables are positively correlated, the assumption of statistical independence results in under-prediction of the overall-model uncertainty. However, as described in the following section, the uncertainties in a few variables almost completely dominated the uncertainty of the simulated DO. These variables most likely have strong correlations between reaches. It may be noted that these variables are also the variables identified as key variables in FORA. In cases where the key variables identified by FORA contribute nearly all the output uncertainty, these results would be unlikely to change if variable correlations were considered (Melching and Yoon 1996).

The other important data necessary for performing the uncertainty analysis (UA) are the base values of all input variables of the model. For determining optimal waste load allocations for dischargers to the system, the values of discharge, DO of headwaters, point load flows and temperature are selected with respect to adverse conditions (e.g., low flow, low DO, high temperature, etc.) prevailing in the river system. For example, a value of 131.75 m^3/s is taken for Tunga River headwater flow, while it's mean value is 166.89 m^3/s. This yields conservative optimal waste load allocation to the dischargers. In the uncertainty analysis, however, the discharge, DO, and temperature are assigned values equal to their mean values given in Tables 2 and 3 to reflect general conditions of the river system. Thus, the mean value of simulated DO in FORA will be equal to the value obtained from the QUAL2E run corresponding to the base values of the input variables. For other variables, (e.g., hydraulic and reaction coefficients, etc.) base values assigned for the determination of optimal fractional removal levels are used in the UA. For solving the S-O, BOD and DO are taken as the pollutant and water quality indicator ($i = n = 1$), respectively. Linear membership functions are considered (α_{il} =1 and β_{imn}=1) for the discharger and PCA goals in constraints (8) and (9). The optimal treatment levels and satisfaction level obtained from S-O are given in Table 3. Both FORA and MCS are done with respect to the optimal fractional removal levels computed from the S-O approach (Table 3).

Table 2. Mean Values for Head Water Flow Conditions

River	River flow (m^3/s)	Dissolved oxygen concentration (mg/L)	Biological oxygen demand concentration (mg/L)
Bhadra	17.80	6.5	1
Tunga	166.89	6.5	1
Kumadavati	14.94	6.5	1
Haridra	13.90	6.0	1

Table 3. Effluent Flow Data and Optimal Fractional Removal Levels

Discharger	Biological oxygen demand concentration (mg/L)	Dissolved oxygen concentration (mg/L)	Effluent flow (m^3/s)	Optimal fractional removal level[a] (%)
D_1	1000	2.0	1.167	74.6
D_2	440	2.0	0.539	74.6
D_3	300	2.0	0.032	66.5
D_4	900	2.0	0.763	35.0
D_5	222	2.0	0.042	35.0
D_6	600	2.0	0.225	35.0
D_7	450	2.0	1.672	35.0
D_8	900	2.0	1.515	45.0

[a]Obtained from fuzzy waste load allocation model: optimal satisfaction level, $\lambda^*=0.28$.

Table 4. Normalized Sensitivity Coefficient Matrix for Dissolved Oxygen Concentration without Incremental Flow

Basic variable	Reach 1 Element 3	Reach 2 Element 3	Reach 3 Element 20	Reach 4 Element 3	Reach 5 Element 2	Reach 7 Element 1	Reach 7 Element 10	Reach 9 Element 19	Reach 11 Element 2	Reach 11 Element 16	Reach 13 Element 1	Reach 13 Element 12	Reach 15 Element 2	Reach 15 Element 19
Reaeration coefficient	0.10	0.15	0.04	0.03	0.05	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Initial temperature	−0.85	−0.93	−0.73	−0.18	−0.70	−0.81	−0.81	−0.79	−0.75	−0.78	−0.80	−0.83	−0.81	−0.87
Head water flow	0.08	0.13	0.00	0.00	0.00	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Head water dissolved oxygen	0.10	0.02	0.06	0.78	0.14	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Point load flow	−0.09	−0.15	−0.02	−0.01	−0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Point load biological oxygen demand	−0.09	−0.16	−0.02	−0.00	−0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10

Table 5. Components of Percentage of Variance Matrix for Dissolved Oxygen Concentrations without Incremental Flow

Basic variable	Reach 1 Element 3	Reach 2 Element 3	Reach 3 Element 20	Reach 4 Element 3	Reach 5 Element 2	Reach 7 Element 1	Reach 7 Element 10	Reach 9 Element 19	Reach 11 Element 2	Reach 11 Element 16	Reach 13 Element 1	Reach 13 Element 12	Reach 15 Element 2	Reach 15 Element 19
Biological oxygen demand decay	4.39	3.55	0.01	0.26	0.41	<1.00	<1.00	<1.00	<1.00	<1.00	0.02	0.43	0.98	2.27
Reaeration coefficient	23.41	30.41	7.64	8.31	11.73	1.83	1.25	0.92	1.27	1.02	0.95	2.05	3.80	12.10
Initial temperature	63.10	49.92	91.43	15.69	86.61	97.93	98.59	98.99	98.48	98.88	98.77	95.82	92.14	80.26
Head water flow	4.14	7.23	0.00	0.01	0.02	<1.00	<1.00	<1.00	<1.00	<1.00	0.07	0.45	0.98	1.76
Head water dissolved oxygen	0.23	0.00	0.13	75.61	0.81	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00
Point load flow	3.78	6.64	0.45	0.08	0.29	<1.00	<1.00	<1.00	<1.00	<1.00	0.15	0.64	1.19	2.36
Point load biological oxygen demand	0.73	1.39	0.09	0.01	0.06	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00
SUM (%)[a]	99.78	99.15	99.75	99.97	99.92	99.76	99.85	99.90	99.75	99.90	99.96	99.40	99.09	98.75
Standard deviation	0.72	0.84	0.55	0.30	0.54	0.60	0.59	0.58	0.55	0.57	0.58	0.60	0.59	0.68

[a]Standard deviation of dissolved oxygen at the location.

8.4.2 Screening of Basic Variables

For carrying out FORA, 14 locations were selected based on the criteria of the lowest DO concentration in a reach and proximity of the locations (checkpoints) to the point loads and junctions. The DO concentrations are the result of a QUAL2E run with respect to the optimal fractional removal levels obtained from the S-O. Details of the locations and their significance are: Locations 1-3, (Reach 1, Computational Element 3), 2-3, 4-3, 7-10 are immediately downstream of point loads, where the lowest DO concentrations of the reach are observed in QUAL2E simulations. Locations 5-2 and 11-2 are downstream of the river junctions of the Tunga-Bhadra and Kumadavati Rivers, respectively. There is no significance to location 7-1, which is merely chosen to learn the effect of the uncertainty in the middle portions of the river. QUAL2E-UNCAS requires the perturbance percentage to the input variable. The NSC and variances are computed for 5% perturbation. The application of a 5% increment in the parameter values was recommended by Brown and Barnwell (1987) for uncertainty calculation in QUAL2E-UNCAS (Melching and Yoon 1996).

Details of the analysis carried out neglecting the incremental flow and nonpoint source pollution is first presented. Table 4 shows the normalized sensitivity coefficient (NSC) matrix for the output variable, DO, obtained from sensitivity analysis. The sensitivity analysis of DO was done for all the basic variables listed in Table 1.The sensitivity analysis has shown that the initial temperature has the highest sensitivity in all reaches except reach 4. The reaeration coefficient, headwater flow, headwater DO, pointload flow and point load BOD have a significant sensitivity (with NSC magnitude greater than 0.1) only at a few locations. An insignificant variable has a sensitivity coefficient equal to zero and near to zero values at all the 14 locations considered in the analysis. For this reason the NSC matrix of Table 4 shows sensitivity coefficients for DO concentration corresponding to only significant variables at the selected locations. QUAL2E-UNCAS performs sensitivity analysis only for 5 locations at a time (i.e., Locations 1-3, 2-3, 3-20, 4-3, 5-2; 7-1, 7-10, 9-19, 11-2, 11-16; and 13-1, 13-12, 15-2, 15-19). During the analysis, if the NSC of any basic variable (e.g., headwater DO in Table 4) is found to be < 0.10 for all the 5 locations, then the NSC is reported as < 0.10 in the coefficient matrix. For this reason, some of the NSC values are reported as < 0.10 in Table 4 at these 5 locations. NSC shows the variables affecting DO concentration. Initial temperature is the predominant variable affecting DO as shown by it having the highest NSC value in all the reaches except in Reach 4. The reaeration coefficient follows temperature in influencing DO with positive relationship although with a very low NSC. Other than these two variables, headwater DO also has an effect on DO, but only in locations 1-3, 4-3 and 5-2. In Reach 4 (Tunga river) headwater flow is considerably higher (166.89 m^3/s)than any other headwater flow and this affects the sensitivity of DO significantly. In middle reaches, DO is invariant to perturbation in variables other than initial temperature and reaeration coefficient. This is possibly due to absence of point loads with high concentrations.

The variance analysis gives the magnitude of variance in the DO concentration due to the variance in an input variable. The contribution in percent of variance of each basic variable to the variance in the DO concentration estimated in QUAL2E-UNCAS with respect to the optimal allocation policy is given in Table 5. Similar to the sensitivity analysis, QUAL2E-UNCAS peforms variance analysis only for 5 locations at a time. During the analysis, if the variance of any basic variable is less than 1% for all 5 locations, the value is reported as < 1% at those locations in Table 5. The results show a similar but a somewhat modified pattern relative to the normalized sensitivity coefficients. As seen from the table, temperature and reaeration coefficient account for more than 95% of the variability in DO concentrations at most locations. DO is more sensitive to temperature as is evident from both NSC matrix and percentage of variance matrix. Some marginal influence of other variables is observed in the last 4 locations (13-1, 13-12, 15-2, and 15-19). Headwater flow, headwater DO and point load BOD have effect both at initial (1-3, 2-3, 3-20, 4-3 and 5-2) and end locations (13-1, 13-12, 15-2 and 15-19). The reasons discussed for NSC are valid here also for the dominance of headwater DO at location 4-3. BOD decay effect is slightly present in the first 2 locations and the last location.

Table 6. Normalized Sensitivity Coefficient Matrix for Dissolved Oxygen Concentration with Incremental Flow

Basic variable	Reach 1 Element 3	Reach 2 Element 3	Reach 3 Element 20	Reach 4 Element 3	Reach 5 Element 2	Reach 7 Element 1	Reach 7 Element 10	Reach 9 Element 19	Reach 11 Element 2	Reach 11 Element 16	Reach 13 Element 1	Reach 13 Element 12	Reach 15 Element 2	Reach 15 Element 19
Biological oxygen demand decay	−0.08	−0.11	−0.02	−0.01	−0.03	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Reaeration coefficient	0.12	0.18	0.09	0.03	0.08	<0.10	<0.10	<0.10	<0.10	<0.10	0.06	0.07	0.08	0.11
Initial temperature	−0.86	−0.98	−0.75	−0.18	−0.72	−0.83	−0.83	−0.81	−0.77	−0.80	−0.82	−0.85	−0.83	−0.89
Head water flow	0.09	0.16	0.04	0.00	0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Head water dissolved oxygen	0.10	0.02	0.06	0.78	0.13	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Point load flow	−0.09	−0.14	−0.02	−0.01	−0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Point load biological oxygen demand	−0.09	−0.15	−0.02	−0.00	−0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10

Table 7. Components of Percentage of Variance Matrix for Dissolved Oxygen Concentrations with Incremental Flow

Basic variable	Reach 1 Element 3	Reach 2 Element 3	Reach 3 Element 20	Reach 4 Element 3	Reach 5 Element 2	Reach 7 Element 1	Reach 7 Element 10	Reach 9 Element 19	Reach 11 Element 2	Reach 11 Element 16	Reach 13 Element 1	Reach 13 Element 12	Reach 15 Element 2	Reach 15 Element 19
Biological oxygen demand decay	5.03	5.21	0.44	0.29	1.01	<1.00	<1.00	<1.00	<1.00	<1.00	0.74	1.62	2.23	3.37
Reaeration coefficient	26.75	36.64	26.59	8.52	20.99	10.16	9.67	11.44	12.12	11.58	11.18	13.67	16.01	26.27
Initial temperature	59.21	43.22	69.17	15.77	75.56	86.88	87.56	84.97	84.43	85.41	85.10	80.44	76.75	64.17
Incremental flow	0.12	0.57	1.73	0.02	0.80	1.05	1.09	1.68	1.61	1.46	1.45	1.39	1.20	1.04
Head water flow	4.66	8.22	1.15	0.00	0.60	0.87	0.79	1.00	0.87	0.73	1.17	1.91	2.37	2.89
Head water dissolved oxygen	0.21	0.00	0.12	75.28	0.60	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00
Point load flow	3.16	4.40	0.26	0.08	0.23	<1.00	<1.00	<1.00	<1.00	<1.00	0.08	0.36	0.66	1.31
SUM (%)[a]	99.15	98.26	99.46	99.96	99.79	98.96	99.11	99.09	99.03	99.18	99.71	99.39	99.22	99.04
Standard deviation	0.74	0.91	0.61	0.30	0.57	0.62	0.62	0.61	0.58	0.60	0.61	0.64	0.63	0.73

[a]Standard deviation of dissolved oxygen at the location.

Table 8. Summary of Monte Carlo Simulation for Dissolved Oxygen (with Incremental Flow)

Reach	Element	Base mean	Simulated mean	Bias	Minimum	Maximum	Standard deviation	Coefficient of variation
1	3	6.61	6.56	−0.05	3.98	7.13	0.28	0.04
2	3	6.17	6.11	−0.06	3.56	6.99	0.40	0.07
3	20	6.85	6.83	0.02	6.08	7.08	0.09	0.01
4	3	6.62	6.62	0.00	5.79	7.47	0.26	0.04
5	2	6.92	6.91	−0.01	5.96	7.16	0.10	0.01
13	1	6.94	6.92	−0.02	6.23	7.09	0.09	0.01
13	12	6.75	6.73	−0.02	5.89	6.98	0.12	0.02
15	2	6.66	6.64	−0.02	5.67	6.93	0.14	0.02
15	19	6.63	6.61	−0.02	5.56	7.01	0.18	0.03

Both NSC and percentage of variance are taken into consideration in screening the basic variables. Melching and Yoon (1996) have indicated that the NSC is not an appropriate way to determine the key parameters. The fraction of variance obtained from Equation (19) is a far more powerful and useful tool. It is seen that only temperature and reaeration coefficient influence DO to a significant extent and headwater flow, headwater DO, and point load BOD to a lesser extent in the river system. Though headwater flow, headwater DO, and point load BOD have influence only at the beginning and end locations, they also have been considered as key random variables in the MCS analysis. Since DO is invariant to all variables except initial temperature and reaeration coefficient at the middle (7-1, 7-10, 9-19, 11-2 and 11-16) locations, these locations are neglected in the MCS analysis.

To examine the effect of nonpoint source pollution, FORA is carried out next by including the incremental flow. Tables 6 and 7 provide the results obtained from sensitivity analysis and FORA, respectively for this case. Except for indicating the incremental flow effect, the FORA results show same trend as in the case of non inclusion of incremental flows. Both the NSC and variance analyses indicate initial temperature and reaeration coefficient as major influencing variables at all the locations. Incremental flow, point load flow, headwater flow and BOD are observed as other influencing variables. Incremental flow has the greatest influence after temperature and reaeration coefficient in all reaches except Reaches 1 and 2. Since the incremental flow is added uniformly, its cumulative magnitude is small in the initial section, Reaches 1 and 2. It may be observed that, at the locations away from point loads, the contribution from variance of incremental flow keeps building up on the downstream side. The point loads in Reaches 11 and 14 change the magnitude and the trend. Based on FORA and NSC results, reaeration coefficient, initial temperature, headwater flow, headwater DO, BOD decay, incremental flow and point load BOD and flow are taken as basic variables. Since DO is invariant to all variables except initial temperature,reaeration coefficient and incremental flow at the middle (7-1, 7-10, 9-19, 11-2 and 11-16) locations, these locations are neglected in the MCS analysis.

8.4.3 Monte Carlo Simulation (MCS) Analysis

Table 8 contains the summary statistics (base mean, simulated mean, bias, minimum, maximum, range, standard deviation, coefficient of variation, and skewness coefficient) for simulated DO (simulated with MCS) concentration at the key locations identified from FORA for analysis neglecting and including the incremental flow respectively. The results show similar trends for the two cases of inclusion and non inclusion of incremental flows. The bias, shown in the tables, is the difference between base value (resulting when the mean values of all parameters are used in the simulation) and simulated mean of DO concentration, whereas range is the difference between the minimum and maximum of all simulated DO concentrations. Initial analysis of MCS, varying the number of simulations has shown that 2000 simulations are sufficient to achieve convergence of the statistics for the simulated DO variable. There is a good match between base and simulated means of DO concentration. Out of all the locations considered for MCS analysis, lowest values of simulated mean and minimum values and highest values of coefficient of variation are observed at location 2-3. This is the most critical location being immediately downstream of two high BOD loads. The trend of the statistical parameters are same at other locations, except for higher magnitudes in simulated mean, minimum, maximum values and lower magnitudes of coefficient of variation.

8.4.4 Evaluation of Fuzzy Risk

The fuzzy risk of low water quality is computed with respect to the output variable DO concentration. Since only one variable is considered for the evaluation of the fuzzy risk, the suffix, i, is dropped. Denoting the fuzzy set of low water quality, DO concentration, and fuzzy risk of low water quality by W_l, c_l, and r_l, respectively, the fuzzy risk is rewritten in discrete form as:

$$r_l = \sum_{c_{min_l}}^{MIN[c_{max_l}, c_l^D]} \mu_{W_l}(c_l)\, p(c_l) \qquad (21)$$

where c_{min_l} and c_{max_l} are the minimum and maximum concentration levels of DO obtained from MCS at checkpoint l. Figure 4 shows a typical membership function of low water quality, $\mu_{W_l}(c_l)$, which is expressed as,

$$\mu_{W_l}(c_l) = [(c_l^D - c_l)/(c_l^D - c_l^L)]^{\gamma_l} \qquad (22)$$

where γ_l is the non zero positive real number defining the shape of the membership function at location l. The value of γ_l may be selected by the decision makers based on their perception of low water quality to a given value of DO. c_l^D is set to 95% of the saturated DO concentration, since an achievement of saturation DO is nearly impossible, even in natural conditions. The value of c_l^L is set to 4 mg/L for the locations in Reaches 1,2 and 3, and 6 mg/L for the other locations. The 95% of saturation and 4 mg/L bounds of the membership function are for illustration purposes only, and a more realistic, ecologically based membership function should be developed in future research. The frequency distribution obtained from MCS is used to compute the probability distribution function $p(c_l)$ in the Equation (21). The membership function, $\mu_{W_l}(c_l)$ of Equation (22) and frequency density function, $p(c_l)$, at key location, l, are substituted in Equation (21). The fuzzy risk is evaluated between c_{min_l} and minimum of c_{max_l} and desirable level c_l^D.

8.4.5 Results and Discussion

Table 9 represents the results of the fuzzy risk levels as well as permissible and desirable DO concentration levels at the selected key locations of the river for three different values of γ. The fuzzy risk trends for the three γ are same. $\gamma < 1$ (0.8) and $\gamma > 1$ (1.2) give higher and lower values compared to linear membership (γ = 1) based values and reflect respectively pessimistic and optimistic perceptions of the decision maker. The results are discussed with reference to the linear membership function. Results for analysis without incremental flow show the highest fuzzy risk at location 4-3. The higher fuzzy risk level at location 4-3 compared to locations 1-3 and 2-3 indicates the effect of minimum permissible and desirable levels. c_l^L at location 4-3 (6 mg/L) is more stringent than that of locations 1-3 and 2-3 (4 mg/L). Setting of 6 mg/L at locations 1-3 and 2-3 yields higher fuzzy risk level at those locations (43.51% and 66.45%) than at location 4-3. Overall, the reason for higher fuzzy risk levels at 1-3, 2-3 and 4-3 is due to their location immediately downstream of high point loads. As the simulated mean DO values at many locations are greater than 6.5 mg/L with a very small variance, the number of simulated DO concentration levels that fall below c_l^L are nil and this results in zero fuzzy risk at those locations.

In Reach 15, the reaeration coefficient is low compared to that in all other reaches. DO is much more sensitive to reaeration in this reach, as reflected in the high variance values for the reaeration coefficient at locations 15-2 and 15-19. This high variance resulted in low values of mean, minimum, and maximum DO values, and high standard deviation of DO at 15-2 and 15-19. These statistics when used in the MCS , result in a higher frequency of DO levels around minimum permissible level. With the fuzzy membership value close to 1 near the minimum permissible value of DO this resulted in a high fuzzy risk at these locations.

Inclusion of incremental flow in the analysis completely alters the trend and magnitudes of the fuzzy risk levels at all locations. In this case, highest fuzzy risk levels are obtained at the last reach due to the obvious reason of cumulative effect of incremental flow and nonpoint source of pollution resulting from it. The fuzzy risk considering the incremental flow may be seen as the sum of fuzzy risk with only point loads and fuzzy risk only with incremental flow. There are no factors other than the incremental flow that change the trend and magnitude of the fuzzy risk levels resulting from only point loads.

The uncertainty analysis is also carried out with the low flow values used (e.g., 131.75 m^3/s for Tunga headwater flow) in deriving the optimal fractional removal levels, with a view to determine the risk under design low flow conditions. The CV of headwater flows for this analysis was determined as 0.27 from the historic mean low flows. The key locations and key variables identified by FORA and sensitivity analysis were the same as those with mean flows. For both cases of neglecting and including incremental flow, the resulting fuzzy risk values at all locations were nearly same as those obtained with mean flows, except at the last two locations, 15-2 and 15-19. At these two locations the fuzzy risk values are 17.64% and 22.58%, respectively for the case of neglecting incremental flow (as against 11.31% and 13.29% with mean flows, shown in Table 11). The fuzzy risk at these two locations, with incremental flow are 46.99% and 52.29% respectively (against, 39.62% and 42.77% with mean flows shown in Table 11).

The crisp risk, defined as $P[c_l \leq c_l^L]$, is also determined at the key locations. As the cumulative frequency of DO concentration level below 6 mg/L is near to zero in the case considering only point loads and low in the case considering incremental flow, the crisp risks are all negligible, being very low to zero. Since the fuzzy risk includes a wider range of DO concentration levels than the crisp risk, in general, the fuzzy risk values will be higher than the crisp risk. As seen from the membership function of low water quality, Figure 4, the fuzzy risk and crisp risk will both be equal to 1 only in the unlikely event of all simulated values of c_l being less than c_l^L.

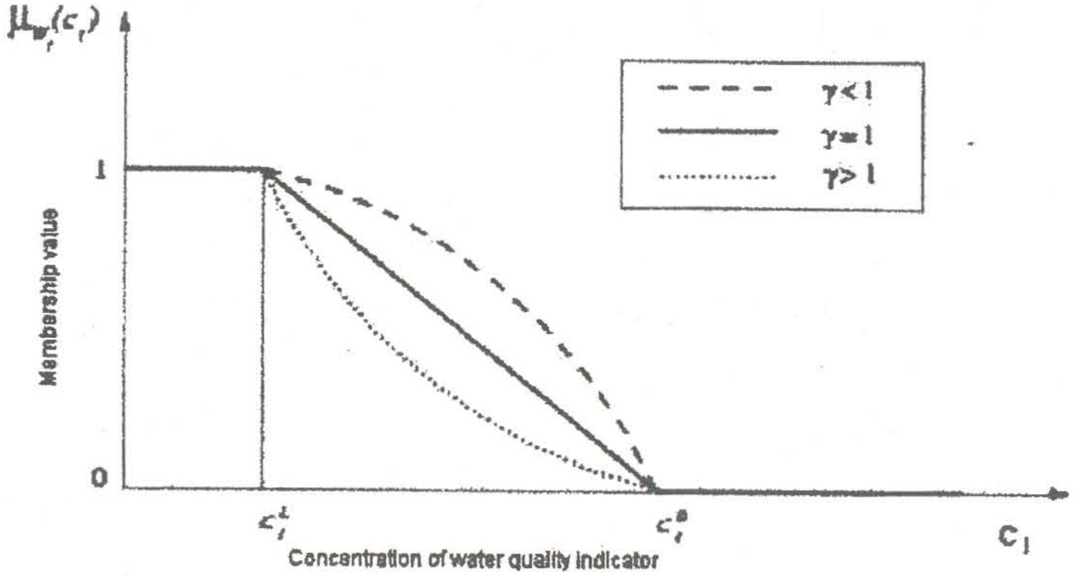

Fig. 4 Membership function for fuzzy set, W_l

8.5 General Remarks

A procedure for risk evaluation in a river water quality control problem is developed in this project. The concept of fuzzy risk is used in the context of water quality-control problems. While the crisp risk denotes the probability of failure, the fuzzy risk indicates the expected degree of failure, and, thus, provides a more general measure of risk. To account for the uncertainty in the standards for determining a failure, occurrence of failure itself has been treated as a fuzzy event, in the work. The fuzzy definition of low water quality ensures that there is no single *threshold* value which defines a failed state. All discrete water quality concentrations have been treated as *failures* of different degrees. The fuzzy set of low water quality maps all water quality levels to *lowwaterquality* and its membership function denotes the degree to which the water quality is low. The membership functions of the fuzzy sets are subjective statements of the perceptions of the decision makers. For example, the membership function for the low water quality indicates the decision maker's perception of the degree of *lowquality*, for a given level of water quality. The lower and upper bounds of the membership functions also are subjective, and in general depend on the particular problem being solved. To address such uncertainty in the lower and upper bounds of the membership functions the fuzzy membership functions themselves may be treated as fuzzy in the model and may be modelled using gray numbers (e.g., Chang et al. 1997). This, however, is not done in this work. It may be noted that allowing the lower limits of the fuzzy membership functions to be less than the normally used standard values such as 5 *mg/L* for DO, and shaping the membership functions with respect to biological information on DO requirements for aquatic life, for example, would be a useful application of the methodology presented.

A fuzzy optimization model is first solved to obtain optimal fractional removal levels, and then, with these optimal fractional removal levels held fixed, the river system is simulated with the input variables treated as random. This implicit approach has the advantage of computational simplicity. Considering the random nature of input variables explicitly in QUAL2E and the optimization model would pose several computational difficulties associated with expressing the stochastic water quality simulation as a set of constraints in the fuzzy optimization model. With a number of variables treated random, it would be impossible to solve a stochastic water quality simulation integrated into the fuzzy optimization model. To overcome these difficulties, the implicit approach is used in this work. The approach, however, has the limitation that the fractional removal levels are determined independent of the uncertainty in the DO concentrations, and, thus, will not be truly optimal. Future research may be directed towards integrating the two models, viz., the optimal fractional removal model, and the risk analysis model to derive policies that minimize the risk of low water quality while maximizing the goal satisfaction.

The optimal fractional removal levels have been obtained from the fuzzy optimization model using critical values of influencing variables (e.g., low stream flow, high temperature, etc.), whereas the entire range of possible values of the variables is used in evaluating the implications of the optimal fractional removal levels. It may be noted that for the purpose of the risk evaluation presented in this report, it is not really necessary to employ FORA. The MCS could just be run with all parameters considered as uncertain at all possible output locations. Doing this, however, would necessitate modification of the QUAL2E code or making multiple UNCAS runs as the number of output locations in QUAL2E is restricted. The key parameters could be determined by a simple regression analysis between the model output and each input parameter. FORA was used in this work, mainly to provide a quick evaluation of the key parameters and key locations.

Table 9. Fuzzy Risk at Key Locations Identified by First Order Reliability Analysis

Location no.	Reach no.	Element no.	Distance from u/s (km)	Minimum permissible level (c_l^L) (mg/L)	Desirable level (c_l^D) (mg/L)	Without incremental flow: Fuzzy risk (r_l) (%) γ=0.8	γ=1.0	γ=1.2	With incremental flow: Fuzzy risk (r_l) (%) γ=0.8	γ=1.0	γ=1.2
1	1	3	3	4.00	7.13	22.35	15.74	11.17	24.88	17.92	13.02
2	2	3	7	4.00	7.15	33.04	25.56	19.91	40.74	32.98	26.84
3	3	20	27	4.00	7.14	0.07	0.03	0.01	15.56	9.85	6.26
4	4	3	3	6.00	7.09	47.51	41.13	35.90	49.22	42.81	37.53
5	5	2	29	6.00	7.09	1.00	0.51	0.27	20.40	14.19	9.96
6	13	1	130	6.00	6.97	0.00	0.00	0.00	0.04	0.02	0.01
7	13	12	141	6.00	6.97	0.89	0.53	0.32	32.86	25.41	19.81
8	15	2	143	6.00	7.06	16.47	11.31	7.88	47.33	39.62	33.31
9	15	19	160	6.00	7.06	18.21	13.29	9.87	50.11	42.77	36.70

8.6 User Manual and Documentation of QUAL2K

A detailed user manual and documentation is prepared for the use of the recently released model, QUAL2K, of the US Environmental Protection Agency (EPA). The purpose of this document is to help users in understanding the procedures for parameter estimation, discretisation schemes to be used in particular cases, interpretation of results and other related details through a case study, so that the software may be used for other case studies also. This document is submitted as a separate document along with this report of the project.

9. Conclusion

In this project, methodologies are developed for addressing uncertainties due to (a) randomness of variables (such as stream temperature, streamflow, friction resistance to flow, effluent flow and concentration, reaction coefficients etc) that influence the water quality in a stream, and (b) fuzziness associated with management goals and imprecision arising out of lack of adequate data. The methodologies are demonstrated with the case study of the Tunga-Bhadra river system in Karnataka, India. In addition, a user manual is prepared for use of the water quality simulation model QUAL2K, the latest model available as a free download in the HEC web page (http://www.epa.gov/athens/wwqtsc/html/qual2k.html). This manual, along with other case study details provided in this report, will be useful for analyising the water quality in a river system in the country.

Publications Resulting from the Work

Subimal Ghosh and P P Mujumdar, (2009) "Fuzzy Waste Load Allocation Model: A Multiobjective Approach" Accepted for publication in *Journal of Hydroinformatics* (Special Edition) (Pub : International Water Association, IWA).

Subimal Ghosh, and P. P. Mujumdar, (2006) "Risk Minimization in Water Quality Control Problems of a River System", *Advances in Water Resources*, Vol. 29, pp. 458-470. (Pub: Elsevier, Netherlands)

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ANNEXURE 1.

STATEMENT OF RECEIPT AND EXPENDITUE
IN RESPECT OF THE SCHEME
FUZZY-STOCHASTIC MODELLING FOR STREAM WATER QUALITY
UPTO: 27.02.2008

Scheme code : MWR0004
Investigator's Name : Dr. P.P.MUJUMDAR
Duration of Scheme : 28/02/2006 to 27/02/2008
Scheme Reference No. : 23/49/2006-R&D/267-79 dtd.23.01.06

Sanction Heads	Amount of Sanction	<..... Expenditure.....> 2005-2006	2006-2007	2007-2008*	Total Expenditure	Sanction Balance
SALARY	2,81,400-00	0-00	76,209-00	1,01,804-00	1,78,013-00	1,03,387-0
EQUIPMENT	40,000-00	0-00	23,176-00	6,552-00	29,728-00	10,272-0
CONTINGENCY	31,100-00	0-00	0-00	28,833-00	28,883-00	2,217-0
EXPERIMENTAL CHARGES	36,000.00	0-00	11,567-00	10,003-00	21,570-00	14,430-0
OVERHEADS	88,500-00	0-00	0-00	88,500-00	88,500-00	0-0
TRAVEL	54,000.00	0-00	18,000-00	33,373-00	51,373-00	2,627-0
TOTAL	5,31,000-00	0-00	1,28,952-00	2,69,115-00	3,98,067-00	1,32,933-0

***PROVISIONAL & SUBJECT TO AUDIT**

Total Receipts : 5,31,000-00
Total Expenditure : 3,98,067-00
Receipts Balance : 1,32,933-00

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